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A Note on Ronald Meek's 'Studies in the Labour Theory of Value' * **

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Abstract: *Ronald Meek has (deliberately) ignored a very important discovery of Jevons. When labour is measured in terms of marginal labour values prices are proportional to these values and commodities exchange accordingly. This has been rediscovered by Soviet economists and that has been published in the JEL in the 70ies. Furthermore it is shown that under neoclassical assumptions the vector of marginal labour values is equal to the Sraffian vector of quantities of dated labour.*

Keywords: Labour Theory of Value; Labour Fund; Euler's Theorem; Marxian Economics; Ronald Meek; Adam Smith; W. Stanley Jevons

JEL Classification: B00, B51, D46

Introductory Remarks to the Marginal Approach to the Labour Theory of Value

The basic proposition of the labour theory of value is the proportionality of the price to the labour necessary for the production of a commodity.

* I owe thanks to John S. Chipman, Stefano Perri and Antoine Rebeyrol for helpful comments and corrections. I only am responsible for remaining errors.

**This paper has been presented at the “Congress Marx International 5” at Université Paris X – Nanterre, October 2007. The following introductory remarks try to outline the logical structure of the problem.

A) Prices without a surplus (simple reproduction)

$$px = wL \quad (1)$$

p - price, x – quantity, w – wage rate, L – labour

The wage rate

$$w = \frac{p}{L/x} \quad (2)$$

Relative prices

$$\frac{p_1}{p_2} = \frac{wL_1/x_1}{wL_2/x_2} = \frac{L_1/x_1}{L_2/x_2} \quad (3)$$

B) Prices with a surplus (capitalistic reproduction)

$$px = wL + (1+r)K \quad (4)$$

r – rate of profit, K – value of capital

Define the rate of surplus value s as well as the organic composition of capital o as

$$s = \frac{rK}{wL} \quad o = \frac{K}{wL} \quad (5)$$

Here we are at a point where further complications arrive as orthodox Marxists claim that the surplus m is independent of capital and the rate of profit. m is a function of labour hours and the wage rate only. Then this $s = m/wL$ has to be transformed into $s = rK/wL$. This is the famous Transformation Problem. The problem has not been resolved properly. But see the works of Gérard Duménil and Duncan Foley on the “New Interpretation”. However if that problem can be resolved we still arrive at the problem as we discuss it here.

Notice the following relation: $r = s/o$

$$r = \frac{s}{o} = \frac{rK/wL}{K/wL} = \frac{rK}{K} \quad (5a)$$

$$p = wL (1 + s + o) \quad (6)$$

Relative prices expressed with rates of surplus value and organic compositions of capital

$$\frac{p_1}{p_2} = \frac{w(1 + s_1 + o_1)L_1/x_1}{w(1 + s_2 + o_2)L_2/x_2} \quad (7)$$

B1) Marxian assumption $s_1 = s_2$

From the assumption follows $\mathbf{o}_1 = \mathbf{o}_2$ as the rate of profit \mathbf{r} is uniform.

$$\frac{p_1}{p_2} = \frac{w(1 + s_1 + o_1)L_1/x_1}{w(1 + s_2 + o_2)L_2/x_2} = \frac{L_1/x_1}{L_2/x_2} \quad (8)$$

Notice that this means that there is a proportionality between paid labour (L_1 and L_2) and price. The problem is $\mathbf{o}_1 = \mathbf{o}_2$

B2) The rates of surplus value are different: $s_1 \neq s_2$

$$\frac{p_1}{p_2} = \frac{w(1 + s_1 + o_1)L_1/x_1}{w(1 + s_2 + o_2)L_2/x_2} = \frac{wL_{e1}/x_1}{wL_{e2}/x_2} = \frac{L_{e1}/x_1}{L_{e2}/x_2} \quad (9)$$

L_{ei} – labour embodied in commodity i

Here we have replaced the expression $(1+s_i+o_i)L_i$ by L_{ei}

The Problem Restated for the Case of Capitalistic Reproduction (see Equations 1 and 2)

If

$$px = wL_e = w \frac{L_e}{x} \quad (10)$$

then

$$w = \frac{p}{L_e/x} \quad (10a)$$

L_e is labour embodied and consists of paid labour, unpaid labour and the labour embodied in the used up capital. Here it is obviously equal to the Ricardian concept of labour commanded, L_c , which is by definition $L_c = p/w$.

The solution of the Marginal Revolution and Jevons (see Jevons' chapter on labour in his 'Theory of Political Economy)

$$\frac{L_e}{x} = \frac{\delta L}{\delta x} = \frac{1}{\delta x / \delta L} \quad (11)$$

and

$$w = \frac{p}{\delta L / \delta x} = p \delta x / \delta L \quad (12)$$

The wage rate is equal to the ratio of price to the marginal labour value or equivalently the wage rate is equal to the value of the marginal product (a standard result in microeconomic theory for the maximization of profits in the case of perfect competition)

The solution substituted into equation (10) gives

$$px = wL_e = w \frac{L_e}{x} x = w \frac{\delta L}{\delta x} x = \frac{p}{\delta L / \delta x} \frac{\delta L}{\delta x} x \quad (13)$$

One should notice that Jevons proves that $\delta L / \delta x$ is **embodied labour**.

The Solution of Sraffa

$$p = wa_n + p(1 + r)A \quad (14)$$

p – vector of prices, a_n – vector of labour coefficients, A – matrix of technical coefficients

$$p[I - (1 + r)A] = wa_n \quad (15)$$

$$p = wa_n[I - (1 + r)A]^{-1} \quad (16)$$

$$v = a_n[I - (1 + r)A]^{-1} \quad (17)$$

$$p = wv \quad (18)$$

Now, the vector v is regarded by the Neo-Ricardians as the vector of vertically integrated labour coefficients if $r = 0$. Its elements indicate the labour hours needed for the production of

a commodity summing up all the labour hours used up also in intermediate production. This can be seen when the vector is expanded to a power series

$$\mathbf{v} = \mathbf{a}_n [I - A]^{-1} = \mathbf{a}_n + \mathbf{a}_n A + \mathbf{a}_n A^2 + \dots \mathbf{a}_n A^{n-1} \quad (19)$$

If the rate of profit r is positive then the Neo-Ricardians speak of quantities of dated labour.

$$\mathbf{v} = \mathbf{a}_n [I - (1 + r)A]^{-1} = \mathbf{a}_n + \mathbf{a}_n (1 + r)A + \mathbf{a}_n (1 + r)^2 A^2 + \dots \mathbf{a}_n (1 + r)^{n-1} A^{n-1} \quad (19)$$

Notice the surplus elements in it the $\mathbf{a}_n r^i A^i$. The Neo-Ricardians do not regard these elements as embodied labour and they speak of the vector \mathbf{v} as representing labour commanded, \mathbf{L}_c . This is because by definition $\mathbf{L}_c = \mathbf{p}/\mathbf{w} = \mathbf{v}$.

But we follow Jevons and regard \mathbf{w} as $w = \frac{p}{\delta L / \delta x}$ This is consistent with Sraffa's argument. Then \mathbf{v} must be

$$\mathbf{v} = \left[\frac{\delta L_1}{\delta x_1}, \frac{\delta L_2}{\delta x_2}, \dots, \frac{\delta L_n}{\delta x_n} \right] \quad (20)$$

With other words Jevons' discovery can justify the position that Sraffian quantities of dated labour are embodied labour. Sraffa's solution should be seen as a reinstatement of the labour theory of value.

Introduction

In his "Studies in the Labour Theory of Value" Ronald Meek has overlooked an important inconsistency in Jevons' treatment of labour. In the chapter on labour in his "Theory of Political Economy" Jevons attributes 2 dimensions to labour: time and intensity (Jevons 1871, p. 170). On the other hand he realizes that in equilibrium utility equals labour and expresses relative prices as the ratio of marginal utilities or marginal productivities (Jevons 1871, p. 186). He discusses labour in terms of disutility and considering the equilibrium situation arrives at the remarkable result "thus we have proved that commodities will exchange in any market in the ratio of the quantities produced by the same quantity of labour" (Jevons, 1871, p. 187)¹.

But surely it is clear that marginal productivity is a function not only of direct labour but also of capital, the value of which can be expressed in labour units. We may best interpret the capital intensity of labour as a crude measure of the social integration of labour, the third dimension of labour². The deeper the social integration of labour the more productive it can be as it is assisted by more embodied labour. Under competitive conditions the higher the capital intensity of labour the more productive must be that labour in order to yield a uniform rate of return on capital.

¹ Ronald Meek does not mention this important remark by Jevons at all in his otherwise so carefully conducted studies. In fact a reference to this statement seems to be very difficult to be found indeed in the literature, Marxian or orthodox.

² Stigler (1941, p.34) criticising Jevons in a footnote suggests also that labour should be considered having three dimensions: "The essential dimensions would be efficiency of labour, duration of labour (per working day), and intensity of labour ..."

Ronald Meek distinguishes two states of the commodity producing society, the early state of simple commodity production and the state of capitalist commodity production. It seems evident that the third dimension of labour, its social integration measured by its capital intensity is exactly the dimension of labour which distinguishes the capitalist state from the simple state of commodity production. Furthermore, instead of invalidating the law of value according to which prices correspond to labour embodied in the commodities, marginal analysis overcomes the famous contradiction between values and prices, prices equal values.

Jevons, instead of refuting Ricardo has in fact made the labour theory of value more perfect although one may render him justice in pointing out that as soon as substitution is allowed production costs are generally not uniquely determined and demand conditions enter into the determination of prices. But in equilibrium commodities exchange according to the labour embodied, labour understood as three-dimensional efficiency units.

Marx (1867) used a more accurate measure of the social integration of labour, the concept of the organic composition of capital which is precisely the ratio of embodied labour in (constant) capital to paid direct labour (wages). Modern economics does not consider wages as capital anymore and it may be more appropriate to name the organic composition of capital *organic composition of labour* instead because that is what it really is. In contrast the capital intensity is a ratio of some physical quantity of a commodity serving as capital and paid direct labour or if the value of capital is used, the money value of capital to paid direct labour. But although Marx had a more accurate measure of the social integration of labour he could not take advantage of it properly in the context of the theory of value. He dealt with the second dimension, intensity, by using the concept of socially necessary labour time and using this as his homogeneous labour unit he had to adapt a unique rate of surplus value.

The Soviet economist A.L. Lur'e developed "An Abstract Model of an Optimal Economic Process and Objectively Determined Valuations" (Lur'e, 1966)³ in which he derives Jevons' solution for the optimal allocation of resources in a planned socialist economy: "in the optimal plan the differential expenditures on various economic resources must be proportional to their differential useful effect for society" (p. 23). In the optimal plan "optimal valuations and the prices that correspond to them will be *proportional* to the socially necessary labor expenditures if by these are meant the marginal increments in labor on the scale of the entire socialist economy corresponding to a unit increase of one or another resource." (p. 27). Lur'e distinguishes in the following *differential* socially necessary expenditures of labour from Marx concept of socially necessary expenditures of labour. His concept of differential socially necessary expenditures of labour corresponds to the vertically integrated labour inputs: "differential expenditures of socially necessary labour refer to both direct and indirect expenditures, reflecting the interrelation of all the links of the national economy that are considered in the preparation and analysis of optimal plans ..." (p. 28) (see also the Appendix).

But Lur'e does not want to interpret his differential socially necessary expenditures of labour or Kantorovich's optimal determined valuations as expressions of Marxian values and refers to Engels' criticism of such interpretations in his preface to the third volume of *Capital*. It

³ This article has been cited in the *Journal of Economic Literature* by Lettiche (1971).

"A comparatively small number of Soviet economists, however, have used mathematical economics as an instrument for theoretical advance and as a stimulus for revision of dogma and obsolete concepts ... His (Lur'es) presentation is definitely *not* representative of the Soviet economic literature in general." (Letiche, 1971, p. 450).

appears to us that Engels' criticism in its theoretical aspects is based on the conviction that one could not derive a logically satisfactory unit of labour values other than in Marx original approach. But it seems to us that Kantorovich and Lur'e have exactly achieved this.

Surely as it stands Marx concept was different. He adapted a unique rate of surplus value independent of the degree of social integration of labour. But is this not a shortcoming of his analysis? We are well able to explain the law of value and the exploitation of labour on the basis of the three-dimensional concept of labour. Engels' criticism of Stiebeling appears as less convincing once one has tackled the mathematical problems and in particular Euler's Theorem. It may be justified to ask if Marx was not busy to improve his mathematical skills after the publication of volume I of *Capital* in order to search for a proof of his theory in terms of marginal analysis. We believe that Engels' criticism of Julius Wolf, G. C. Stiebeling and others should be understood first of all as politically motivated. As Ronald Meek puts it: "Marxists have indeed opposed the numerous suggestions which have been made from both inside and outside their ranks to purge the labour theory from the body of Marxism, or to "reconcile" it with the marginal utility theory ... They have been encouraged in this view by the fact that many of those within their own ranks who have criticised the labour theory have eventually shown themselves to be interested not so much in purging the labour theory from Marxism as in purging Marxism itself from the ideology of the labour movement." (Meek, 1976, p. 202).

A modern economist who is a case in point is John S. Chipman who has discussed the problem of the unique rate of surplus value. Chipman (1952) poses the question "whether any consistent theory of value can be drawn from Marx's writings. It will be shown that one assumption – namely that of equal rates of surplus value among industries – is responsible for

nearly all the errors and contradictions in Marx's theory; and that this assumption must be altogether discarded if the theory is to uphold any claim to empirical validity." (p. 527).

Chipman (referring to *Capital*, vol. III, ch. X) criticises Marx justification of equal rates of surplus value: "Marx believed that competition tended to equalize rates of surplus value. The mechanism by which such equality is achieved is not described; apparently it is competition among workers. There does not seem to be any valid reason why competition among workers, or among firms, should equalize rates of surplus value." (Chipman 1952, p. 533 f.). He comes to the conclusion "The rate of surplus value is not independent of the organic composition of capital, but is, on the contrary, directly related to it. ... Prices, then, are everywhere equal to values, and individual profits are equal to individual surplus values, and the Marxian contradiction is easily solved." (p. 540 f.)

One should think that here Chipman has found his way to *differential socially necessary expenditures of labour* and the *marginal labour theory of value*. But in his conclusions he scraps it altogether: "[Marx] wished, whenever he thought he could get away with it, to cling assiduously to the labour theory of value. Even where he had to reject it, he took pains to point out that whenever a discrepancy in value could not be imputed to labor, labor was still its 'source', though another factor provided its 'natural basis'". (p. 552).

Marginal analysis can best take as a starting point Adam Smith's labour fund approach. "The annual labour of every nation is the *fund* which originally supplies it with all the necessities and conveniences of life ..." (Smith 1776, p.1).

Commonly the concept of the labour fund refers to direct paid labour inputs only. Our interest is to find the appropriate mathematical expression of 'labour embodied' which is a much

wider concept including also unpaid direct labour as well as indirect labour, the labour stored-up in capital goods. This ‘labour embodied’, L_e , which we may call the *effective Labour Fund*, can be expressed as a function of the quantities of the commodities in which the labour is embodied.

$$L_e = f(x_1, x_2, \dots, x_n) \quad (1)$$

Where L_e is labour embodied and x_1, x_2, \dots, x_n are the quantities of commodities 1 to n.

As a first approach we should assume that doubling the amount of all commodities increases the labour embodied by the same proportion which implies constant returns to scale in the production of the commodities. Constant returns to scale are necessary to assure perfect competition (see Meek 1973, p. 74). Under such conditions the function is homogeneous of degree one and using Euler’s Theorem one obtains labour embodied as the sum of the marginal labour costs of the commodities.

$$L_e = \frac{\delta L}{\delta x_1} x_1 + \frac{\delta L}{\delta x_2} x_2 + \dots + \frac{\delta L}{\delta x_n} x_n \quad (2)$$

In fact the marginal labour value approach does not require constant returns to scale in the production technologies.

Equally we may obtain labour embodied by dividing direct paid labour inputs of each industry by the production elasticities of the industries which is

$$L_e = \sum_{i=1}^n \frac{L_i}{a_i} = \sum_{i=1}^n \frac{L_i}{(\delta x_i / \delta L)(L_i / x_i)} = \sum_{i=1}^n \frac{\delta L}{\delta x_i} x_i \quad (3)$$

where L_i is the direct paid labour input of industry i , $a_i = (\delta x_i / \delta L)(L_i / x_i)$ is the production elasticity of labour of industry i . Here we see that all that is required is that the production

function is continuous and everywhere differentiable and invertible i.e. there is an inverse of the function.

Interesting is the relation between ‘labour embodied’, L_e and ‘labour commanded’. Labour commanded L_c is defined as the ratio of the value of output, Y to the unique wage rate w .

If the wage rate equals the value of the marginal product that is, if

$$w = p_i \frac{\delta x_i}{\delta L}; \quad (i=1, \dots, n) \quad (4)$$

p_i – the price of commodity i

labour embodied equals labour commanded as multiplication of labour embodied with the wage rate equals the money value of the commodity $p_i x_i$.

It is

$$Y = w L_e = w \sum_{i=1}^n (\delta L / \delta x_i) x_i = \sum_{i=1}^n (\delta x_i / \delta L) (\delta L / \delta x_i) x_i = \sum_{i=1}^n p_i x_i \quad (5)$$

Y – value of output in terms of money

from which follows that

$$Y / w = L_c = L_e \quad (6)$$

If the wage rate w is smaller than the value of the marginal product (neoclassical exploitation)

$$w < p_i \frac{\delta x_i}{\delta L}; \quad (i=1, \dots, n) \quad (7)$$

labour commanded is greater than labour embodied as it is the case of a monopoly.

If the wage rate is greater than the marginal product (progressive economic (wage) policy)

$$w > p_i \frac{\delta x_i}{\delta L} ; (i = 1, \dots, n) \quad (8)$$

labour commanded is smaller than labour embodied.

One may ask further what relative prices are if commodities exchange according to labour units embodied in them (Jevons discovery, see page 1).

If two commodities exchange according to the labour units embodied in them:

$$\frac{\delta L}{\delta x_1} x_1 = \frac{\delta L}{\delta x_2} x_2 \quad (9)$$

and

$$x_1 = \frac{\delta L / \delta x_2}{\delta L / \delta x_1} x_2 \quad (10)$$

Relative prices are:

$$\frac{p_2}{p_1} = \frac{\delta L / \delta x_2}{\delta L / \delta x_1} = \frac{\delta x_1 / \delta L}{\delta x_2 / \delta L} \quad (11)$$

The last expression means that relative prices are equal to the reciprocal of their marginal productivities. But this is the necessary condition for the optimal allocation of resources which is:

$$p_1 (\delta x_1 / \delta L) = p_2 (\delta x_2 / \delta L) \quad (12)$$

We see that the labour fund approach is fully consistent with modern price theory.

Abandoning the unique rate of surplus value does not necessitate the abandonment of the labour theory of value as Chipman maintains. Marginal analysis and Euler's Theorem which Marx did not know, enables one to derive homogeneous units of labour. One might speak indeed of *differential socially necessary expenditures of labour*.

One should seek the development of the labour theory of value into a *marginal labour theory of value* based on Adam Smith's labour fund as indicated above. So it can become that pillar of modern economic theory which concentrates on the production relations of the economic process. In this way Alfred Marshall's view of the continuity in the development of economic theory becomes much truer than Marshall himself had envisaged it to be.

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Appendix: The Vector of Labour Costs

In his book “Lectures on the Theory of Production” Luigi Pasinetti introduces the concept of *the vector of vertically integrated labour coefficients*

$$\mathbf{v} = \mathbf{a}_n[\mathbf{I} - \mathbf{A}]^{-1} \quad (1)$$

The meaning of this concept is limited to the case where the rate of profit is zero. I argue that it is appropriate to interpret the vector

$$\mathbf{v} = \mathbf{a}_n[\mathbf{I} - (1+\pi)\mathbf{A}]^{-1} \quad (2)$$

as *the vector of labour costs* also in the general case of a positive rate of profit π .

I want to put the point more clearly. Let's suppose that there is an economy with continuous production functions in all of its $n-1$ sectors. In equilibrium the allocation of its resources is optimal. One can still describe and analyse this economy in terms of linear algebra and describe it with $[\mathbf{A}, \mathbf{a}_n]'$ although this is not "really" the technique of the system. We then arrive at equation (V.3.1a) p. 73 in the "Lectures",

$$\mathbf{p}\mathbf{A}(1+\pi) + \mathbf{a}_n\mathbf{w} = \mathbf{p} \quad (3)$$

and this can be written as

$$\mathbf{p} = \mathbf{a}_n[\mathbf{I} - (1+\pi)\mathbf{A}]^{-1}\mathbf{w} \quad (\text{V.5.18}) \text{ p. 80} \quad (4)$$

My point now is that under the assumptions above the row vector

$$\mathbf{v} = \mathbf{a}_n[\mathbf{I} - (1+\pi)\mathbf{A}]^{-1} \quad (5)$$

is equal to

$$\mathbf{v} = \mathbf{a}_n[\mathbf{I} - (1+\pi)\mathbf{A}]^{-1} = [\delta\mathbf{L}/\delta\mathbf{Q}_1, \dots, \delta\mathbf{L}/\mathbf{Q}_{n-1}] \quad (6)$$

where $\delta\mathbf{L}/\delta\mathbf{Q}_i$ is the marginal labour costs of sector i .

That this must be so can easily be shown. If labour is optimally allocated the uniform wage rate is equal to the value of the marginal product of each sector.

$$\mathbf{w} = \mathbf{p}_i \delta\mathbf{Q}_i/\delta\mathbf{L} \quad \text{for } i = 1, \dots, n-1 \quad (7)$$

We can write equation (4) as

$$\mathbf{p} = \mathbf{a}_n[\mathbf{I} - (1+\pi)\mathbf{A}]^{-1}\mathbf{w}\mathbf{I} \quad (4a)$$

$\mathbf{w}\mathbf{I}$ is a diagonal matrix with the wage rate on its major diagonal. We replace the wage rate for each sector by its value of the marginal product $\mathbf{p}_i \delta\mathbf{Q}_i/\delta\mathbf{L}$ and call that matrix \mathbf{W} so that our equation (4a) becomes

$$\mathbf{p} = \mathbf{a}_n[\mathbf{I} - (1+\pi)\mathbf{A}]^{-1}\mathbf{W} \quad (4b)$$

Now it is evident that the elements of $\mathbf{a}_n[\mathbf{I} - (1+\pi)\mathbf{A}]^{-1}$ must be the marginal labour costs as in (6) to cancel out with the marginal productivities of \mathbf{W} to yield the price vector \mathbf{p} .

Finally I would like to point out that the product of the vector \mathbf{v} which should be called **the vector of labour costs**, the product of that vector with the vector of total outputs \mathbf{q} yields what I have called *the effective labour fund*.

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